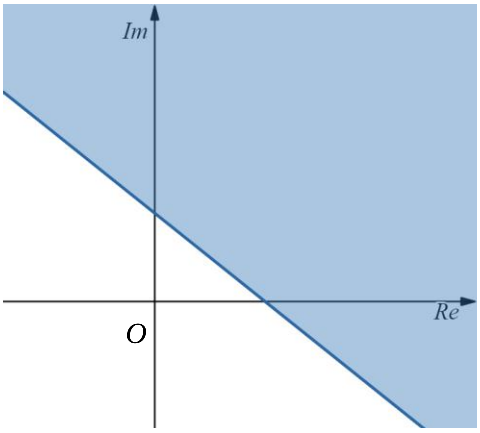
Pearson Edexcel

Mark Scheme (Results)

Summer 2024

Pearson Edexcel International Advanced Level in Further Pure Mathematics (WFM02) Paper 01

Question Number	Scheme	Marks
1(a)	$\lvert z-3-4\mathrm{i}\rvert = \lvert z+1+\mathrm{i}\rvert \Rightarrow \lvert x+\mathrm{i}y-3-4\mathrm{i}\rvert = \lvert x+\mathrm{i}y+1+\mathrm{i}\rvert$ $(x-3)^2+(y-4)^2=(x+1)^2+(y+1)^2$	M1
	$x^2-6x+9+y^2-8y+16=x^2+2x+1+y^2+2y+1 \Rightarrow ax+by+c=0$	dM1
	$8x+10y-23=0$	A1
		(3)
(a) ALT	Mid-point $\left(\dfrac{3-1}{2},\dfrac{4-1}{2}\right)$ and $m=\dfrac{4+1}{3+1}$	M1
	$y-\dfrac{3}{2}=-\dfrac{4}{5}(x-1) \Rightarrow ax+by+c=0$	dM1
	$8x+10y-23=0$	A1
(b)		B1
		(1)
		Total 4

Notes:

(a)

M1: Introduces $z = x + \mathrm{i}y$ (or with other variables) and attempts a correct modulus method – the i's should be dealt with correctly but allow for e.g. minor slips in notation with them before applying Pythagoras. Allow sign slips in the i coordinates, e.g. $(x-3)^2+(y\pm4)^2=(x+1)^2+(y\pm1)^2$ Must be + between the brackets.

dM1: Expands and attempts the required form.

A1: This equation or any integer multiple.

Alternative: via perpendicular bisector

M1: Uses the points (3, ±4) and (–1, ±1) to find the midpoint and gradient between them.

dM1: Correct attempt at the equation of the perpendicular bisector and rearranges to the required form. Must be using negative reciprocal of the gradient between points (but watch for those who go directly to this).

A1: This equation or any integer multiple.

(b) May be scored independently of (a) – using the perpendicular bisector.

B1: Shades the area above an appropriate line, must be in quadrants 1,2 and 4. Accept with solid, dashed or dotted line. Allow for any line with negative gradient and positive y intercept. Do not be concerned about labelling. Part (a) need not have been correct.

Question Number	Scheme	Marks
2(a)	$x\frac{\mathrm{d}y}{\mathrm{d}x}-y^3=4\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}+x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-3y^2\frac{\mathrm{d}y}{\mathrm{d}x}=0$ Alt: $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{y^3}{x}+\frac{4}{x}\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\frac{y^3}{x^2}+\frac{3y^2}{x}\frac{\mathrm{d}y}{\mathrm{d}x}-\frac{4}{x^2}$	M1A1
	$\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+x\frac{\mathrm{d}^3y}{\mathrm{d}x^3}+\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-6y\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2-3y^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=0$ Alt: $\Rightarrow\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{2y^3}{x^3}-\frac{3y^2}{x^2}\frac{\mathrm{d}y}{\mathrm{d}x}-\frac{3y^2}{x^2}\frac{\mathrm{d}y}{\mathrm{d}x}+\frac{1}{x}\left(6y\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+3y^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)+\frac{8}{x^3}$	dM1
	$\Rightarrow x\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=6y\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+\left(3y^2-2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$	A1
		(4)
(b)	$x=2,\ y=1\Rightarrow\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{x=2}=\frac{4+1}{2}=\frac{5}{2},\ \left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_{x=2}=\frac{3(1)\left(\frac{5}{2}\right)-\frac{5}{2}}{2}=\frac{5}{2}$ $\left(\frac{\mathrm{d}^3y}{\mathrm{d}x^3}\right)_{x=2}=\frac{6(1)\left(\frac{25}{4}\right)+\frac{5}{2}}{2}=20$	M1
	$(y=)\mathrm{f}(2)+(x-2)\mathrm{f}'(2)+\frac{(x-2)^2}{2}\mathrm{f}''(2)+\frac{(x-2)^3}{6}\mathrm{f}'''(2)+\ldots$ $=1+\frac{5}{2}(x-2)+\frac{5}{2}\frac{(x-2)^2}{2}+20\frac{(x-2)^3}{6}+\ldots$	M1
	$(y=)1+\frac{5}{2}(x-2)+\frac{5}{4}(x-2)^2+\frac{10}{3}(x-2)^3+\ldots$	A1
		(3)
		Total 7

Notes: Accept other derivative notations, e.g. *y*’, throughout.

(a)

M1: $x\frac{\mathrm{d}y}{\mathrm{d}x}\rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}+x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ or $y^3\rightarrow ky^2\frac{\mathrm{d}y}{\mathrm{d}x}$ Alternatively for $\frac{y^3}{x}\rightarrow\pm\alpha\frac{y^3}{x^2}+\frac{ky^2}{x}\frac{\mathrm{d}y}{\mathrm{d}x}$ (oe via quotient rule), or any correct attempt at implicit differentiation on the equation (coefficient slips condoned).

A1: Any fully correct second derivative expression. The two most common are shown in the scheme.

dM1: Differentiates again using the product rule correctly at least once. Depends on the first method mark. Again, look for correct forms, but allow if coefficients are incorrect.

There may be alternatives where they substitute for $\frac{\mathrm{d}y}{\mathrm{d}x}$ before differentiating again.

The scheme follows the same principle. If unsure, use the Review system.

A1: Correct answer achieved from correct work. In the Alt they need to substitute the second derivative expression before reaching the final answer.

(b)

M1: Attempts to find values for the first 3 derivatives at $x = 2$. Working need not be seen, in which case accept if all are attempted with at least *y*’ correct providing expressions involving the other derivatives were actually found in part (a).

M1: Correct application of Taylor’s theorem with their values up to at least the $(x-2)^2$ term.

A1: Correct simplified expression. May be called *y* or f(*x*) or unlabelled for this part.

Question Number	Scheme	Marks
3(a)	$\dfrac{1}{(n+3)(n+5)} \equiv \dfrac{A}{(n+3)} + \dfrac{B}{(n+5)}$ $\Rightarrow A = \ldots, B = \ldots$	M1
	$\dfrac{1}{2(n+3)} - \dfrac{1}{2(n+5)}$ (oe)	A1
		(2)
(b)	$\left(\dfrac{1}{2}\right)\sum_{r=1}^{n}\left(\dfrac{1}{r+3} - \dfrac{1}{r+5}\right) =$ $\left(\dfrac{1}{2}\right)\left(\dfrac{1}{4} - \dfrac{1}{6} + \dfrac{1}{5} - \dfrac{1}{7} + \dfrac{1}{6} - \dfrac{1}{8} + \ldots + \dfrac{1}{n+2} - \dfrac{1}{n+4} + \dfrac{1}{n+3} - \dfrac{1}{n+5}\right)$	M1
	$= \left(\dfrac{1}{2}\right)\left(\dfrac{1}{4} + \dfrac{1}{5} - \dfrac{1}{n+4} - \dfrac{1}{n+5}\right)$ (oe)	A1
	$= \left(\dfrac{1}{2}\right)\left(\dfrac{9(n+4)(n+5) - 20(n+5) - 20(n+4)}{20(n+4)(n+5)}\right) = \ldots$	dM1
	$= \dfrac{n(9n+41)}{40(n+4)(n+5)}$	A1
		(4)
(c)	$\dfrac{1}{9\times 11} + \dfrac{1}{10\times 12} + \ldots + \dfrac{1}{24\times 26} = \dfrac{21(9\times 21+41)}{40\times 25\times 26} - \dfrac{5(9\times 5+41)}{40\times 9\times 10}$	M1
	$= \dfrac{194}{2925}$	A1
		(2)
		Total 8

Notes:

(a)

M1: Correct partial fraction attempt to obtain values for *A* and *B*. May be implied by correct values.

A1: Correct expression as shown or any equivalent. May be ½ in each numerator, or the ½ may be factored out. The expression must be given in (a) not just embedded in the summation in (b).

(b)

M1: Uses the method of differences to identify non-cancelling terms. Expect to see at least first two pairs and last pair of terms listed, but this may be implied by the correct first two or last two non-cancelling terms being stated with no cancelling explicitly shown. Allow if the ½ is missing for this mark.

A1: Identifies the correct non-cancelling terms. Allow if the ½ is missing for this mark.

dM1: Puts over a common denominator $k(n+4)(n+5)$, where $k \neq 1$, and makes progress towards the required form. The ½ may be missing for this mark. Allow slips in the numerator coefficients but should have correct terms in *n*.

A1: Achieves the correct answer.

(c)

M1: Attempts to use the formula from (b) with $\sum_{r=1}^{21} - \sum_{r=1}^{k}$ where $k = 5$ or 6.

Substitution into the formula must be seen at least once.

A1: Correct answer achieved (the M must have been scored). Must be simplified.

Question Number	Scheme	Marks
4(a)	E.g. $2y\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{t^2}\frac{\mathrm{d}t}{\mathrm{d}x}$ or $y^2=\frac{1}{t}\Rightarrow t=y^{-2}\Rightarrow\frac{\mathrm{d}t}{\mathrm{d}x}=-2y^{-3}\frac{\mathrm{d}y}{\mathrm{d}x}$ or $y=t^{-\frac{1}{2}}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\times\frac{\mathrm{d}t}{\mathrm{d}x}=-\frac{1}{2}t^{-\frac{3}{2}}\frac{\mathrm{d}t}{\mathrm{d}x}$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x}+y=xy^3\Rightarrow-\frac{t^{-\frac{3}{2}}}{2}\frac{\mathrm{d}t}{\mathrm{d}x}+\frac{1}{\sqrt{t}}=xt^{-\frac{3}{2}}$	M1
	$\frac{\mathrm{d}t}{\mathrm{d}x}-2t=-2x$ *	A1*
		(3)
(b)	$I=\mathrm{e}^{-\int 2\,\mathrm{d}x}=\mathrm{e}^{-2x}$	B1
	$t\mathrm{e}^{-2x}=-2\int x\mathrm{e}^{-2x}\,\mathrm{d}x$	M1
	$=x\mathrm{e}^{-2x}-\int \mathrm{e}^{-2x}\,\mathrm{d}x$	M1
	$=x\mathrm{e}^{-2x}+\frac{1}{2}\mathrm{e}^{-2x}\ (+c)$	A1
	$t=\frac{1}{y^2}=x+\frac{1}{2}+c\mathrm{e}^{2x}$ $\Rightarrow y^2=\frac{1}{x+\frac{1}{2}+c\mathrm{e}^{2x}}\left(\text{or e.g. }\frac{2}{2x+1+A\mathrm{e}^{2x}}\right)$ (oe)	M1A1
		(6)
		Total 9

Notes:

(a)

B1: Any appropriate correct first derivative statement connecting $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}t}{\mathrm{d}x}$. May be implied by use of chain rule when substituting.

M1: Having attempted a first derivative statement, substitutes completely into the given differential equation to obtain an equation in x and t only (which may be the final step).

A1*: Achieves the printed answer with no errors.

(b)

B1: Correct integrating factor. May be implied by working.

M1: For $\text{their } IF\times t=\text{"}-2\text{"}\int x\times\text{their } IF\ (\mathrm{d}x)$ provided their IF is a function in x.

M1: Integrates RHS by parts to obtain $\alpha x\mathrm{e}^{\pm kx}+\beta\int \mathrm{e}^{\pm kx}\ (\mathrm{d}x)$

A1: Correct RHS (not follow through – must be the correct RHS). The + c is not needed for this mark.

M1: Reverses the substitution and makes y^2 the subject. The constant of integration must have been introduced at the right time and been treated correctly, although the algebra in rearranging need not be fully correct.

A1: Correct answer in the required form with the constant correctly placed. Does not need to be fully simplified, and e.g. "2c" is fine for the constant and allow for "2c" renamed and "c" and such. ISW after a correct answer is seen.

Alt (b)	$m-2=0 \Rightarrow m=2$ so CF is $t=A\mathrm{e}^{2x}$	B1
	For PI try $t=ax+b$	M1
	$\frac{\mathrm{d}t}{\mathrm{d}x}-2t=-2x \Rightarrow a-2(ax+b)=-2x \Rightarrow a=..,b=...$	M1
	$a=1, b=\frac{1}{2} \Rightarrow t=A\mathrm{e}^{2x}+x+\frac{1}{2}$	A1
	$t=\frac{1}{y^2}=x+\frac{1}{2}+A\mathrm{e}^{2x}$ $\Rightarrow y^2=\frac{1}{x+\frac{1}{2}+A\mathrm{e}^{2x}}\left(\text{or e.g.}\frac{2}{2x+1+B\mathrm{e}^{2x}}\right)$ (oe)	M1A1
		(6)

Notes:

B1: Solves the auxiliary equation and forms the correct complementary function.

M1: Selects an appropriate form for the particular integral.

M1: Differentiates and substitutes into the differential equation to find values for the constants of the PI.

A1: Correct values and adds the CF and PI (t = may be missing for this mark).

M1: Reverses the substitution and makes y^2 the subject. Must have formed an equation using **both** PI and CF for this mark.

A1: Correct answer in the required form with the constant correctly placed. Does not need to be fully simplified and same conditions for main scheme. ISW after a correct answer is seen.

Question Number	Scheme	Marks
5	$x=-2,\ 3$	B1
	$\dfrac{x+1}{(x-3)(x+2)}=1-\dfrac{2}{x-3}\Rightarrow x+1=x^2-x-6-2(x+2)\Rightarrow x^2-4x-11=0$ **or** $\dfrac{x+1}{(x-3)(x+2)}-1+\dfrac{2}{x-3}\leqslant 0\Rightarrow\dfrac{4x-x^2+11}{(x-3)(x+2)}=0$ **or** $(x+1)(x-3)(x+2)\leqslant(x-3)^2(x+2)^2-2(x-3)(x+2)^2$ $\Rightarrow(x-3)(x+2)\left[4x-x^2+11\right]=0$	M1
	$x^2-4x-11=0\Rightarrow x=\dfrac{4\pm\sqrt{16+44}}{2}=...$	M1
	$x=2\pm\sqrt{15}$	A1
	$x<-2,\ 2-\sqrt{15}\leqslant x<3,\ x\geqslant 2+\sqrt{15}$	M1A1
		(6)
		Total 6

Notes:

B1: The cv's −2 and 3 stated or used. May be seen on a diagram.

M1: Any complete **algebraic** method to reach at least a expanded **quadratic** for the other two critical values. E.g. equates sides, multiplies through and simplifies, or collects to one side and finds common denominator or multiplies through by a positive expression and collects to one side etc. Do not be concerned about inaccuracies in the inequalities in intermediate steps – score for the overall method leading to a suitable equation being found.

M1: Correct method (usual rules) to solve the resulting **three term quadratic** equation for the remaining cv's. Once a quadratic is reached accept if correct solutions for their quadratic appear without working.

A1: For the cv's $2\pm\sqrt{15}$ Must be exact.

M1: Uses their cv's form the correct shape for the solution, $x<\alpha,\ b\leqslant x<\gamma,\ x\geqslant\delta$ allowing for the inequalities to be loose or strict in each case. Number lines, sign tables or graph may be used, or this may be deduced from inequalities work throughout the question.

A1: Fully correct and no extra other region. Must be exact values. Accept interval notation, e.g $(-\infty,-2)\cup\left[2-\sqrt{15},3\right)\cup\left[2+\sqrt{15},\infty\right)$. Note if set notation is used the "union" symbol must be correct, but in language accept "and" or "or".

Note: Solutions that expand to a quartic and then state the four CVs with no algebraic method shown to solve the quartic score at most SC B1M0M0A0M1A1.

Question Number	Scheme	Notes	Marks
6 **Way 1**	$w=\frac{z-\mathrm{i}}{z+1}\Rightarrow z=\frac{w+\mathrm{i}}{1-w}$ (oe)		B1
	$z=\frac{w+\mathrm{i}}{1-w}=\frac{u+\mathrm{i}v+\mathrm{i}}{1-u-\mathrm{i}v}\times\frac{1-u+\mathrm{i}v}{1-u+\mathrm{i}v}$		M1
	$z=\frac{u-u^2+uv\mathrm{i}-v\mathrm{i}-uv\mathrm{i}+\mathrm{i}-u\mathrm{i}-v^2-v+\mathrm{i}v+\mathrm{i}}{(1-u)^2+v^2}$ $\mathrm{Re}(z)=0\Rightarrow u-u^2-v^2-v=0$		M1A1
	$u-u^2-v^2-v=0\Rightarrow\left(u-\frac{1}{2}\right)^2+\left(v+\frac{1}{2}\right)^2=\frac{1}{2}$		dM1
	Centre : $\left(\frac{1}{2},-\frac{1}{2}\right)$ Radius : $\frac{1}{\sqrt{2}}$		A1A1
			(7)
Way 2	$w=\frac{z-\mathrm{i}}{z+1}\Rightarrow w=\frac{\mathrm{i}y-\mathrm{i}}{\mathrm{i}y+1}$		B1
	$w=\frac{\mathrm{i}y-\mathrm{i}}{\mathrm{i}y+1}\times\frac{1-\mathrm{i}y}{1-\mathrm{i}y}$		M1
	$u+\mathrm{i}v=\frac{\mathrm{i}y+y^2-\mathrm{i}-y}{1+y^2}=\frac{y^2-y}{1+y^2}+\frac{y-1}{1+y^2}\mathrm{i}\Rightarrow\frac{u}{v}=y$ $u=\frac{\frac{u^2}{v^2}-\frac{u}{v}}{1+\frac{u^2}{v^2}}$ or $v=\frac{\frac{u}{v}-1}{1+\frac{u^2}{v^2}}$ $\left(u-u^2-v^2-v=0\right)$		M1A1
	$u-u^2-v^2-v=0\Rightarrow\left(u-\frac{1}{2}\right)^2+\left(v+\frac{1}{2}\right)^2=\frac{1}{2}$		dM1
	Centre : $\left(\frac{1}{2},-\frac{1}{2}\right)$ Radius : $\frac{1}{\sqrt{2}}$		A1A1

Notes:

B1: Correct rearrangement as shown or equivalent.

M1: Introduces $w = u + \mathrm{i}v$ (or may use other variables) and multiplies numerator (and denominator) by the conjugate of the denominator.

M1: Expands the numerator and sets the real part to zero – allow if there are slips in the denominator e.g. wrong sign.

A1: Correct equation in any form.

dM1: Completes the square for u and v. They must have attained an expression on which they need to complete the square on at least on of u or v. Alternatively allow for correct centre up to sign error, or correct radius, deduced from expanded circle equation.

A1: Correct centre – but must be from a circle equation. Accept as coordinates or as $\frac{1}{2}-\frac{1}{2}\mathrm{i}$ and be tolerant of notation errors, e.g. $\left(\frac{1}{2},-\frac{1}{2}\mathrm{i}\right)$ if the correct point is clear.

A1: Correct radius following a correct equation in u and v (allowing sign slips on u and v). Must be exact, but accept equivalents.

Alternative:

B1: Replaces z with iy (oe), or may replaces by x +iy and later set x = 0 in the equation.

M1: Multiplies numerator and denominator by the conjugate of the denominator.

M1: Rearranges into real and imaginary parts to obtain y in terms of u and v and substitutes into u or v to eliminate y. Allow recovery from slips in the denominator per main scheme.

A1: Correct equation in any form.

dM1: Completes the square for u and v. They must have attained an expression on which they need to complete the square on at least on of u or v. Alternative as per main scheme.

A1: Correct centre. Accept as coordinates or as $\frac{1}{2}-\frac{1}{2}\text{i}$ and be tolerant of notation errors, e.g. $\left(\frac{1}{2},-\frac{1}{2}\text{i}\right)$ if the correct point is clear.

A1: Correct radius from correct work (allowing sign slips on u and v). Must be exact, but accept equivalents.

Note: Some rarer alternatives may be seen, but if you are unsure of a method send to review.

Way 3	$0\to -\text{i}, \text{i}\to 0, \infty\to 1$	B1
	E.g. Bisector of 0 and 1 is $u=\frac{1}{2}$	M1
	Bisector of 0 and −i is $v=-\frac{1}{2}$	M1A1
	Bisectors meet at when $u=\frac{1}{2}$ and $v=-\frac{1}{2}$	dM1
	Centre : $\left(\frac{1}{2},-\frac{1}{2}\right)$ Radius : $\frac{1}{\sqrt{2}}$	A1A1

Notes:

B1: Identifies three correct image points on the circle.

M1: Finds at least two image points on the circle and attempts the perpendicular bisector of at least one pair of points. Alt: attempts to substitute into a general circle equation and makes some progress eliminating one variable.

M1: Finds three image points and attempts two perpendicular bisectors. Alt: uses the third point to form a second equation in the same two variables.

A1: Correct bisectors. Alt: correct equations.

dM1: Finds the intersection of the bisectors. Note MMAM may be implied in this method if the bisectors are obvious (as with the three points shown in the scheme). Alt: valid method for fully solving the simultaneous equations.

A1: Correct centre. Accept as coordinates or as $\frac{1}{2}-\frac{1}{2}\text{i}$ and be tolerant of notation errors, e.g. $\left(\frac{1}{2},-\frac{1}{2}\text{i}\right)$ if the correct point is clear.

A1: Correct radius from correct work (allowing sign slips on u and v). Must be exact, but accept equivalents.

Way 4: Using inverse points of the imaginary axis, e.g. ± 1 is also possible. Here as −1 maps to ∞ so 1 maps to the centre of the circle, and then any other point may be used to work out the radius of the circle. If other inverse points are used it will be more complicated. Send to review if you see any responses like this that you feel deserve merit.

Question Number	Scheme	Marks
7(a)	$y = \mathrm{e}^x \sin x \Rightarrow \left(\frac{\mathrm{d}y}{\mathrm{d}x} =\right) \mathrm{e}^x \sin x + \mathrm{e}^x \cos x$ $\Rightarrow \left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2} =\right) \mathrm{e}^x \sin x + \mathrm{e}^x \cos x + \mathrm{e}^x \cos x - \mathrm{e}^x \sin x = 2\mathrm{e}^x \cos x$ (oe – see notes)	M1
	E.g. (see notes for equivalents) $\Rightarrow \left(\frac{\mathrm{d}^3y}{\mathrm{d}x^3} =\right) 2\mathrm{e}^x \cos x - 2\mathrm{e}^x \sin x$ $\Rightarrow \frac{\mathrm{d}^4y}{\mathrm{d}x^4} = 2\mathrm{e}^x \cos x - 2\mathrm{e}^x \sin x - 2\mathrm{e}^x \cos x - 2\mathrm{e}^x \sin x$ $\frac{\mathrm{d}^4y}{\mathrm{d}x^4} = -4\mathrm{e}^x \sin x = -4y$ (oe – see notes)	dM1 A1
	$\Rightarrow \frac{\mathrm{d}^4y}{\mathrm{d}x^4} = -4y \Rightarrow \frac{\mathrm{d}^5y}{\mathrm{d}x^5} = -4\frac{\mathrm{d}y}{\mathrm{d}x} \Rightarrow \frac{\mathrm{d}^6y}{\mathrm{d}x^6} = -4\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$	A1
		(4)
(b)	$(y)_0 = 0, \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 = 1, \left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_0 = 2, \left(\frac{\mathrm{d}^3y}{\mathrm{d}x^3}\right)_0 = 2, \left(\frac{\mathrm{d}^4y}{\mathrm{d}x^4}\right)_0 = 0, \left(\frac{\mathrm{d}^5y}{\mathrm{d}x^5}\right)_0 = -4, \left(\frac{\mathrm{d}^6y}{\mathrm{d}x^6}\right)_0 = -8$	M1
	$(y =) 0 + x + \frac{x^2}{2}\times 2 + \frac{x^3}{3!}\times 2 + 0 - \frac{x^5}{5!}\times 4 - \frac{x^6}{6!}\times 8 + \ldots$	M1
	$(y =) x + x^2 + \frac{x^3}{3} - \frac{x^5}{30} - \frac{x^6}{90} + \ldots$	A1
		(3)
		Total 7

Notes: Accept other forms of notation for the derivatives, e.g. *y*' etc.

(a)

M1: Applies the product rule on *y* and *y*' to reach at least the second derivative. Condone mislabelling of the derivatives for the Ms. May work in terms of *x* or *y* or a combination of both, e.g. $y' = y + \mathrm{e}^x \cos x, y'' = y' + \mathrm{e}^x \cos x - \mathrm{e}^x \sin x = y' - y + \mathrm{e}^x \cos x = 2y' - 2y$ etc

dM1: Continues the differentiation process to obtain an expression for the 4th derivative. Again, may be working in either *x* or *y* or a combination of both.

A1: Any correct fourth derivative expression, may be unsimplified and award when first seen, e.g. may identify $\frac{\mathrm{d}^4y}{\mathrm{d}x^4} = -4y$ or even $2y''' - 2y''$. Must be correctly identified as the fourth derivative.

A1: Completes the process to obtain a correct expression, or for $k = -4$. May work in *x* or *y* or a mixture for the final derivatives.

E.g. $y^{vi} = 2y^{v} - 2y^{iv} = 4y^{iv} - 4y''' - 2y^{iv} = 2y^{iv} - 4y''' = 4y''' - 4y'' - 4y''' = -4y''$ is one possible variation, but others are possible with earlier eliminations. Most will use main scheme.

(b)

M1: Attempts to find values for all 6 derivatives at $x = 0$, but condone one missing as a slip. Accept values seen stated for each derivative as an attempt.

M1: Applies Maclaurin's theorem correctly with their values, allowing for one slip in the factorials, up to the term in x^6 (or a fifth non-zero term if errors mean the x^6 terms is not needed).

A1: Correct expansion, condone missing the "$y =$".

Question Number	Scheme	Marks
8(a)	E.g. $\frac{dy}{dx}=\frac{dy}{dt}\frac{dt}{dx}=\frac{1}{x}\frac{dy}{dt}$ or $t=\ln x\Rightarrow x=e^t\Rightarrow\frac{dx}{dy}=e^t\frac{dt}{dy}$ or $t=\ln x\Rightarrow\frac{dt}{dy}=\frac{1}{x}\frac{dx}{dy}$ etc.	B1
	E.g. $\frac{d^2y}{dx^2}=-\frac{1}{x^2}\frac{dy}{dt}+\frac{1}{x}\frac{d^2y}{dt^2}\frac{dt}{dx}$ or $\frac{dx}{dy}=e^t\frac{dt}{dy}\Rightarrow\frac{dy}{dx}=e^{-t}\frac{dy}{dt}\Rightarrow\frac{d^2y}{dx^2}=-e^{-t}\frac{dt}{dx}\frac{dy}{dt}+e^{-t}\frac{d^2y}{dt^2}\frac{dt}{dx}$ or $x\frac{dy}{dx}=\frac{dy}{dt}\Rightarrow x\frac{d^2y}{dx^2}+\frac{dy}{dx}=\frac{d^2y}{dt^2}\frac{dt}{dx}$ etc	M1
	E.g. $\left(t=\ln x\Rightarrow\frac{dt}{dx}=\frac{1}{x}\Rightarrow\right)\frac{d^2y}{dx^2}=-\frac{1}{x^2}\frac{dy}{dt}+\frac{1}{x}\frac{d^2y}{dt^2}\frac{dt}{dx}=-e^{-2t}\frac{dy}{dt}+e^{-2t}\frac{d^2y}{dt^2}$ or $\left(x=e^t\Rightarrow\frac{dx}{dt}=e^t\Rightarrow\right)\frac{d^2y}{dx^2}=-e^{-t}\frac{dt}{dx}\frac{dy}{dt}+e^{-t}\frac{d^2y}{dt^2}\frac{dt}{dx}=-e^{-2t}\frac{dy}{dt}+e^{-2t}\frac{d^2y}{dt^2}$ or $x\frac{d^2y}{dx^2}+\frac{dy}{dx}=\frac{d^2y}{dt^2}\frac{dt}{dx}\Rightarrow e^t\frac{d^2y}{dx^2}+\frac{1}{e^t}\frac{dy}{dt}=\frac{1}{e^t}\frac{d^2y}{dt^2}$ $\Rightarrow\frac{d^2y}{dx^2}=e^{-2t}\left(\frac{d^2y}{dt^2}-\frac{dy}{dt}\right)$*	A1*
		(3)
(b)	$x^2\frac{d^2y}{dx^2}-2y=1+4\ln x-2(\ln x)^2\Rightarrow e^{2t}\times e^{-2t}\left(\frac{d^2y}{dt^2}-\frac{dy}{dt}\right)-2y=1+4t-2t^2$ $\Rightarrow\frac{d^2y}{dt^2}-\frac{dy}{dt}-2y=1+4t-2t^2$ *	B1*
		(1)
(c)	$m^2-m-2=0\Rightarrow m=2,\ -1$ $y=Ae^{2t}+Be^{-t}$	M1 A1
	$y=at^2+bt+c\Rightarrow\frac{dy}{dt}=2at+b\Rightarrow\frac{d^2y}{dt^2}=2a$	M1
	$\Rightarrow 2a-2at-b-2at^2-2bt-2c=1+4t-2t^2$ $2a=2\Rightarrow a=1$ $-2a-2b=4\Rightarrow b=-3$ $2a-b-2c=1\Rightarrow c=2$	dM1
	$y=Ae^{2t}+Be^{-t}+t^2-3t+2$	A1
		(5)
(d)	$y=Ax^2+Bx^{-1}+(\ln x)^2-3\ln x+2$	B1ft
		(1)
		Total 10

Notes:

(a)

B1: Any correct equation linking $\frac{dy}{dx}$ and $\frac{dy}{dt}$ (or their reciprocals).

M1: Differentiates again wrt x or t with attempts at both the product rule **and** chain rule to obtain an equation involving $\frac{d^2y}{dx^2}$ and $\frac{d^2y}{dt^2}$. Allow slips as long as an attempt at both rules is made.

A1*: Eliminates the x's and $\frac{dx}{dt}$'s from the equation and completes to obtain the printed answer with no errors.

(b)

B1*: Shows a clear substitution into the equation and proceeds to the given answer with no incorrect steps seen. Working may be minimal as long as there is at least one step before the given answer that shows substitution has been made.

(c)

M1: Forms and attempts (need not be a correct attempt) to solve $m^2 - m - 2 = 0$ and attempts to form the appropriate complementary function for their roots. Allow if variables are incorrect for the methods (ie *x* used instead of *t*).

A1: Correct complementary function. The *y* =… may be missing. Must be in terms of *t*.

M1: Starts with the correct form the particular integral and differentiates twice.

dM1: Substitutes into the correct differential equation and solves to find at least two of *a* and *b* and *c* by comparing coefficients. Do not be concerned about the method of solving, look for any attempt reaching values for at least two of the constants.

A1: Correct solution. Must start *y* = … . Allow if stated in (d).

(d)

B1ft: Correct equation with exponentials simplified to powers of *x* (follow through their answer to (c)) and isw if incorrect simplification takes place after a correct answer seen. Should start *y* = … accept if GS =, or similar, is used for **both** (c) and (d). Accept $\ln^2 x$ for $(\ln x)^2$ but not $\ln x^2$.

Note: parts (c) and (d) may be merged, but to award the A mark for (d) a correct general solution must be given. Allow the mark in (d) if seen in part (c).

Question Number	Scheme	Marks
9(a)	$(\cos 6\theta =)\text{Re}(\cos\theta + \text{i}\sin\theta)^6 = \cos^6\theta - 15\cos^4\theta\sin^2\theta + 15\cos^2\theta\sin^4\theta - \sin^6\theta$	M1A1
	$\cos^6\theta - 15\cos^4\theta\sin^2\theta + 15\cos^2\theta\sin^4\theta - \sin^6\theta$ $= \cos^6\theta - 15\cos^4\theta(1-\cos^2\theta) + 15\cos^2\theta(1-\cos^2\theta)^2 - (1-\cos^2\theta)^3$	M1
	$= \cos^6\theta - 15\cos^4\theta + 15\cos^6\theta + 15\cos^2\theta(1 - 2\cos^2\theta + \cos^4\theta) - (1 - 3\cos^2\theta + 3\cos^4\theta - \cos^6\theta)$ $\cos 6\theta \equiv 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$*	A1*
		(4)
(b)	$48x^6 - 72x^4 + 27x^2 - 1 = 0$ $x = \cos\theta \Rightarrow \frac{3}{2}\cos 6\theta + \frac{1}{2} = 0 \Rightarrow \cos 6\theta = ...\left(-\frac{1}{3}\right)$	M1
	$\cos 6\theta = -\frac{1}{3} \Rightarrow 6\theta = ... \Rightarrow \theta = ...$ (pv 6θ =109.5°, 1.91 rad, θ = 18.25°, 0.318 rad)	dM1
	$x = \cos\theta = ...$ (commonly 0.950)	dM1
	$x = 0.204$	A1
		(4)
		Total 8

Notes:

(a)

M1: Attempts the binomial expansion of $(\cos\theta + \text{i}\sin\theta)^6$ and extracts the real terms. Binomial coefficients should be correct but allow if there are sign errors. Ignore errors on imaginary terms. Allow use of $(c+\text{i}s)^6$ notation.

A1: Correct extracted expression. Need not be set equal to cos 6θ at this stage.

M1: Applies $\sin^2\theta = 1 - \cos^2\theta$ to obtain an expression in cos θ only.

A1*: Sets equal to cos 6θ and fully expands the square and cube brackets (must be shown) then completes with no errors to obtain the given answer.

(b)

Note: The question says hence and thus methods via solving the cubic in x^2 and square rooting the smallest root score no marks.

M1: Attempts to use the result from part (a) and obtains $\cos 6\theta = k,\ |k| < 1,\ k \neq 0$

dM1: Solves to obtain at least one value for θ. If the $6\theta = \arccos\left(-\frac{1}{3}\right)$ is seen, accept any value that follows for the attempt, but if this method is not shown answers must be correct for their value of $\cos(6\theta)$. The principal values are shown in the scheme. Other values that may be seen are
In radians: for 6θ 4.33, 8.19, 10.66, 14.48, 16.94 ; for θ 0.73, 1.37, 1.78, 2.41, 2.82
In degrees: for 6θ 251°, 469°, 611°, 829°, 1331°; for θ 41.8°, 78.2°, 102°, 138°, 222°;

dM1: Depends on first M. Reverses the substitution to find at least one value for x.
Full list of values for reference: 0.950, 0.746, 0.204, −0.204, −0.746, −0.950

A1: Awrt 0.204 and must be seen identified as the answer, not just given in a list.

Note those who find $\cos 6\theta = -\frac{1}{3}$ but go on to find $6\theta = \arccos\frac{1}{3} = 1.23$ (70.5°), and so $\theta = 0.205$ (11.8°) etc (0.979 as primary answer) may score the M1M0M1A0 but watch out for those who use $\pi - \arccos\frac{1}{3}$ as these can score full marks.

9(a) Alt	$\left(z+\frac{1}{z}\right)^6=z^6+6z^4+15z^2+20+\frac{15}{z^2}+\frac{6}{z^4}+\frac{1}{z^6}$ $=\left(z^6+\frac{1}{z^6}\right)+6\left(z^4+\frac{1}{z^4}\right)+15\left(z^2+\frac{1}{z^2}\right)+20$ $=2\cos 6\theta+6(2\cos 4\theta)+15(2\cos 2\theta)+20$	M1A1
	$=2\cos 6\theta+12\left(2\cos^2 2\theta-1\right)+30\left(2\cos^2\theta-1\right)+20$ $=2\cos 6\theta+12\left(2\left(2\cos^2\theta-1\right)^2-1\right)+30\left(2\cos^2\theta-1\right)+20$	M1
	$64\cos^6\theta=2\cos 6\theta+12\left(8\cos^4\theta-8\cos^2\theta+1\right)+60\cos^2\theta-30+20$ $\Rightarrow \cos 6\theta\equiv 32\cos^6\theta-48\cos^4\theta+18\cos^2\theta-1$*	A1*
		(4)

Notes:
(a)

M1: Attempts the binomial expansion of $\left(z+z^{-1}\right)^6$, groups terms (may be implied) and attempts to replace the z^n+z^{-n} to achieve an expression in cosines. Allow if there are errors in the "$2\cos(n\theta)$".

A1: Correct expression in cosines. Need not be set equal to $64\cos^6\theta$ at this stage.

M1: Applies $\cos 2A=2\cos^2 A-1$ repeatedly to write the $\cos 4\theta$ and $\cos 2\theta$ in terms of $\cos\theta$ only.

A1*: Sets equal to $64\cos^6\theta$, expands all brackets and rearranges to the given expression with no errors to obtain the given answer.

Question Number	Scheme	Marks
10(a)	$1+\cos\theta = 1+\dfrac{\sqrt{3}}{2} \Rightarrow \cos\theta = \dfrac{\sqrt{3}}{2}$ $\Rightarrow 1+\dfrac{\sqrt{3}}{2} = k\times\dfrac{2}{\sqrt{3}} \Rightarrow k = ...$	M1
	$k = \dfrac{\sqrt{3}}{2}+\dfrac{3}{4}$ oe	A1
		(2)
(b)	At P $\theta = \dfrac{\pi}{6}$	B1
	$\int (1+\cos\theta)^2 \, d\theta = \int \left(1+2\cos\theta+\cos^2\theta\right) d\theta$ $= \int \left(1+2\cos\theta+\dfrac{1}{2}+\dfrac{1}{2}\cos 2\theta\right) d\theta$	M1
	$\int (1+\cos\theta)^2 \, d\theta = \dfrac{3}{2}\theta + 2\sin\theta + \dfrac{1}{4}\sin 2\theta$ (oe)	A1
	$\dfrac{1}{2}\left[\dfrac{3}{2}\theta + 2\sin\theta + \dfrac{1}{4}\sin 2\theta\right]_{"\frac{\pi}{6}"}^{\pi}$ $= \dfrac{1}{2}\left[\left(\dfrac{3\pi}{2}+0+0\right) - \left(\dfrac{\pi}{4}+1+\dfrac{\sqrt{3}}{8}\right)\right] \quad \left(= \dfrac{5\pi}{8} - \dfrac{\sqrt{3}}{16} - \dfrac{1}{2}\right)$	M1
	Triangle: $\dfrac{1}{2}\times\left(1+\dfrac{\sqrt{3}}{2}\right)\sin\dfrac{\pi}{6}\times\left(1+\dfrac{\sqrt{3}}{2}\right)\cos\dfrac{\pi}{6} = \dfrac{1}{2}\left(\dfrac{1}{2}+\dfrac{\sqrt{3}}{4}\right)\left(\dfrac{\sqrt{3}}{2}+\dfrac{3}{4}\right)$ $\left(= \dfrac{7\sqrt{3}}{32} + \dfrac{3}{8}\right)$	M1
	Area of $R = \dfrac{5\pi}{8} - \dfrac{\sqrt{3}}{16} - \dfrac{1}{2} + \dfrac{7\sqrt{3}}{32} + \dfrac{3}{8}$	dM1
	$\dfrac{5\pi}{8} + \dfrac{5\sqrt{3}}{32} - \dfrac{1}{8}$	A1
		(7)
		Total 9

Notes:

(a)

M1: Correct method to find the value of k.

A1: Correct value.

(b)

B1: Deduces the correct value of θ at P. May be seen anywhere – in (a) or on Figure 1, or as a limit on their integral.

M1: Attempts $\left(\dfrac{1}{2}\right)\int r^2 \, d\theta$ achieve (at least) three terms from squaring and applies $\cos^2\theta = \pm\dfrac{1}{2}\pm\dfrac{1}{2}\cos 2\theta$. The $\dfrac{1}{2}$ in the area formula may be omitted for this mark.

Caution: Many have learned a formula $\int \cos^2\theta \, d\theta = \dfrac{\theta}{2}+\dfrac{1}{4}\sin 2\theta$ so the use of the double angle formula may be implied by this.

A1: Correct integration of the r^2

M1: Applies the limits of their $\frac{\pi}{6}$ and π to their attempt at the integral (there must be some evidence of the attempt to subtract). Alternatively, may use limits 0 to π **and** 0 to their $\frac{\pi}{6}$, then subtract. Not dependent, so allow if incorrect double angle identity was used or only two terms from squaring.

M1: Uses a correct strategy to find the exact for the area of the triangle. This may be via $\frac{1}{2}ab$ or $\frac{1}{2}ab\sin C$ as shown in scheme (both are equivalent) or by integration $\frac{1}{2}\int_0^{"\pi/6"} k^2\sec^2\theta\,\mathrm{d}\theta = \frac{k^2}{2}\left[A\tan\theta\right]_0^{"\pi/6"} = \ldots$ Accept $\int_0^{\pi/6}\sec^2\theta\,\mathrm{d}\theta = \frac{\sqrt{3}}{3}$ (oe) without further working shown.

dM1: Fully correct method for the required area. Depends on all previous method marks. The $\frac{1}{2}$ in the area formula must have been included to score this mark.

A1: Cao. Terms may be in a different order and accept if put over a common denominator.